## Boolean Retrieval ${ }^{1}$

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${ }^{1}$ Vorlage: Folien von M. Schütze zu [1]

## Boolean Retrieval

■ Fragen sind Boolesche Ausdrücke, z.B.: Caesar AND Brutus

■ Die Suchmaschine gibt alle Dokumente zurück, die der Anfrage entsprechen.

## Does Google use the Boolean model?

## Outline

1 Inverted index

## 2 Processing Boolean queries

## Unstructured data in 1650

- Which plays of Shakespeare contain the words Brutus AND Caesar, but NOT Calpurnia?
- One could grep all of Shakespeare's plays for Brutus and Caesar, then strip out lines containing Calpurnia.
- Why is grep not the solution?


## Unstructured data in 1650

- Which plays of Shakespeare contain the words Brutus AND Caesar, but NOT Calpurnia?
- One could grep all of Shakespeare's plays for Brutus and Caesar, then strip out lines containing Calpurnia.
- Why is grep not the solution?
- Slow (for large collections)
- " NOT Calpurnia" is non-trivial
- Other operations (e.g., find the word Romans near countryman) not feasible
- Ranked retrieval (best documents to return) - focus of later lectures, but not this one


## Term-document incidence matrix

| Anthony | Julius | The | Hamlet | Othello | Macbeth |
| :---: | :---: | :---: | :---: | :---: | :---: |
| and | Caesar | Tempest |  |  |  |


| Anthony | 1 | 1 | 0 | 0 | 0 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Brutus | 1 | 1 | 0 | 1 | 0 | 0 |
| Caesar | 1 | 1 | 0 | 1 | 1 | 1 |
| Calpurnia | 0 | 1 | 0 | 0 | 0 | 0 |
| Cleopatra | 1 | 0 | 0 | 0 | 0 | 0 |
| mercy | 1 | 0 | 1 | 1 | 1 | 1 |
| worser | 1 | 0 | 1 | 1 | 1 | 0 |

Entry is 1 if term occurs. Example: Calpurnia occurs in Julius Caesar. Entry is 0 if term doesn't occur. Example: Calpurnia doesn't occur in The tempest.

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## Incidence vectors

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- To answer the query Brutus AND Caesar AND NOT Calpurnia:
- Take the vectors for Brutus, Caesar, and Calpurnia
- Complement the vector of Calpurnia
- Do a (bitwise) AND on the three vectors
- 110100 AND 110111 AND $101111=100100$


## $0 / 1$ vector for Brutus

|  | Anthony <br> and <br> Cleopatra | Julius <br> Caesar | The <br> Tempest | Hamlet | Othello | Macbeth | $\ldots$ |
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## Answers to query

Anthony and Cleopatra, Act III, Scene ii
Agrippa [Aside to Domitius Enobarbus]: Why, Enobarbus, When Antony found Julius Caesar dead, He cried almost to roaring; and he wept When at Philippi he found Brutus slain.

Hamlet, Act III, Scene ii
Lord Polonius:

I did enact Julius Caesar: I was killed i' the Capitol; Brutus killed me.

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- Assume there are $M=500,000$ distinct terms in the collection
- (Notice that we are making a term/token distinction.)


## Can't build the incidence matrix

- $M=500,000 \times 10^{6}=$ half a trillion 0 s and 1 s .
- But the matrix has no more than one billion 1 s .
- Matrix is extremely sparse.

■ What is a better representations?

- We only record the 1 s .


## Inverted Index

For each term $t$, we store a list of all documents that contain $t$.

| Brutus |
| :---: | | 1 | 2 | 4 | 11 | 31 | 45 | 173 | 174 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| Caesar $\longrightarrow$1 2 4 5 6 16 57 132 $\ldots$ $\mathbf{l}$ |
| :---: |

Calpurnia $\longrightarrow$| 2 | 31 | 54 | 101 |
| :--- | :--- | :--- | :--- | dictionary

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dictionary

## Inverted index construction

1 Collect the documents to be indexed:
Friends, Romans, countrymen.
So let it be with Caesar

2 Tokenize the text, turning each document into a list of tokens: | Friends Romans countrymen So... |
| :--- | :--- | :--- |

3 Do linguistic preprocessing, producing a list of normalized tokens, which are the indexing terms: friend roman countryman so ...
4 Index the documents that each term occurs in by creating an inverted index, consisting of a dictionary and postings.

## Tokenization and preprocessing

Doc 1. I did enact Julius Caesar: I was killed i' the Capitol; Brutus killed me.
Doc 2. So let it be with Caesar. The noble Brutus hath told you Caesar was ambitious:

Doc 1. I did enact julius caesar I was killed i' the capitol brutus killed me Doc 2. so let it be with caesar the noble brutus hath told you caesar was ambitious

## Generate postings



## Sort postings

| term | doclD | term doclD |  |
| :--- | ---: | :--- | ---: |
| l | 1 | ambitious | 2 |
| did | 1 | be | 2 |
| enact | 1 | brutus | 1 |
| julius | 1 | brutus | 2 |
| caesar | 1 | capitol | 1 |
| l | 1 | caesar | 1 |
| was | 1 | caesar | 2 |
| killed | 1 | caesar | 2 |
| i | 1 | did | 1 |
| the | 1 | enact | 1 |
| capitol | 1 | hath | 1 |
| brutus | 1 | 1 | 1 |
| killed | 1 | 1 | 1 |
| me | 1 | i | 1 |
| so | 2 | it | 2 |
| let | 2 | julius | 1 |
| it | 2 | killed | 1 |
| be | 2 | killed | 1 |
| with | 2 | let | 2 |
| caesar | 2 | me | 1 |
| the | 2 | noble | 2 |
| noble | 2 | so | 2 |
| brutus | 2 | the | 1 |
| hath | 2 | the | 2 |
| told | 2 | told | 2 |
| you | 2 | you | 2 |
| caesar | 2 | was | 1 |
| was | 2 | was | 2 |
| ambitious | 2 | with | 2 |
|  |  |  |  |

## Create postings lists, determine document frequency



## Split the result into dictionary and postings file

| Brutus | $\rightarrow$1 2 4 11 31 45 173 174 <br> Caesar        |
| ---: | :--- |
| Calpurnia $\rightarrow$1 2 4 5 6 16 2 31 54 101 |  |

postings file

## Outline

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- To find all matching documents using inverted index:


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3 Locate Calpurnia in the dictionary

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## Simple conjunctive query（two terms）

■ Consider the query：Brutus AND Calpurnia
－To find all matching documents using inverted index：
1 Locate Brutus in the dictionary
2 Retrieve its postings list from the postings file
3 Locate Calpurnia in the dictionary
4 Retrieve its postings list from the postings file
5 Intersect the two postings lists

## Simple conjunctive query（two terms）

■ Consider the query：Brutus AND Calpurnia
－To find all matching documents using inverted index：
1 Locate Brutus in the dictionary
2 Retrieve its postings list from the postings file
3 Locate Calpurnia in the dictionary
4 Retrieve its postings list from the postings file
5 Intersect the two postings lists
6 Return intersection to user

## Intersecting two postings lists



Intersection $\Longrightarrow$

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## Intersecting two postings lists

Brutus $\longrightarrow 0 \rightarrow 2 \rightarrow 4 \rightarrow 11 \rightarrow \sqrt{31} \rightarrow 45 \rightarrow \boxed{173} \rightarrow \boxed{174}$
Calpurnia $\longrightarrow 2 \rightarrow 31 \rightarrow 54 \rightarrow 101$
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- This is linear in the length of the postings lists.
- This only works if postings lists are sorted.

Intersecting two postings lists

```
\(\operatorname{INTERSECT}\left(p_{1}, p_{2}\right)\)
1 answer \(\leftarrow\rangle\)
2 while \(p_{1} \neq\) NIL and \(p_{2} \neq\) NIL
3 do if \(\operatorname{docl} D\left(p_{1}\right)=\operatorname{docID}\left(p_{2}\right)\)
\[
p_{1} \leftarrow \operatorname{next}\left(p_{1}\right)
\]
\[
p_{2} \leftarrow \operatorname{next}\left(p_{2}\right)
\]
\[
\text { else if } \operatorname{docID}\left(p_{1}\right)<\operatorname{docID}\left(p_{2}\right)
\]
7 else if \(\operatorname{docID}\left(p_{1}\right)<\operatorname{docID}\left(p_{2}\right)\) then \(p_{1} \leftarrow \operatorname{next}\left(p_{1}\right)\) else \(p_{2} \leftarrow \operatorname{next}\left(p_{2}\right)\)
```

10 return answer

## Boolean queries

－The Boolean retrieval model can answer any query that is a Boolean expression．
－Boolean queries are queries that use AND，OR and NOT to join query terms．
－Views each document as a set of terms．
－Is precise：Document matches condition or not．
■ Primary commercial retrieval tool for 3 decades
■ Many professional searchers（e．g．，lawyers）still like Boolean queries

■ You know exactly what you are getting．
－Many search systems you use are also Boolean：email，intranet etc．

## Query optimization

- What is the best order for query processing?

■ Consider a query that is an AND of $n$ terms, $n>2$
■ For each of the terms, get its postings list, then AND them together
■ Example query: Brutus AND Calpurnia AND Caesar

## Query optimization

■ Example query: Brutus AND Calpurnia AND Caesar

Brutus $\rightarrow 4 \rightarrow 2 \rightarrow 4 \rightarrow 11 \rightarrow 31 \rightarrow 45 \rightarrow 173 \rightarrow 174$
Calpurnia $\longrightarrow 2 \rightarrow 31 \rightarrow 54 \rightarrow 101$
Caesar $\longrightarrow 5 \rightarrow 31$

## Query optimization

■ Example query：Brutus AND Calpurnia AND Caesar
－Simple and effective optimization：Process in order of increasing frequency
－Start with the shortest postings list，then keep cutting further
■ In this example，first Caesar，then Calpurnia，then Brutus
Brutus $\rightarrow 1 \rightarrow 2 \rightarrow 4 \rightarrow 11 \rightarrow 31 \rightarrow 45 \rightarrow 173 \rightarrow 174$
Calpurnia $\rightarrow 2 \rightarrow 31 \rightarrow 54 \rightarrow 101$
Caesar $\longrightarrow 5 \rightarrow 31$

Optimized intersection algorithm for conjunctive queries

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    4
    5
    6
    7 else if \(\operatorname{docID}\left(p_{1}\right)<\operatorname{docID}\left(p_{2}\right)\)
        8
        9
        then \(\operatorname{ADD}\left(\right.\) answer, \(\left.\operatorname{docID}\left(p_{1}\right)\right)\)
            \(p_{1} \leftarrow \operatorname{next}\left(p_{1}\right)\)
            \(p_{2} \leftarrow \operatorname{next}\left(p_{2}\right)\)
                then \(p_{1} \leftarrow \operatorname{next}\left(p_{1}\right)\)
                else \(p_{2} \leftarrow \operatorname{next}\left(p_{2}\right)\)
```

10 return answer

## More general optimization

- Example query: ( madding OR crowd) AND (ignoble OR strife)
- Get frequencies for all terms

■ Estimate the size of each OR by the sum of its frequencies (conservative)
■ Process in increasing order of OR sizes

## Exercise

Recommend a query processing order for: ( tangerine OR trees) AND ( marmalade OR skies) AND (kaleidoscope OR eyes)

围 H. S. Christopher Manning, P. Raghavan. Introduction to Information Retrieval. Cambridge, 2008.

